2019-2020 In-Class Test Solutions

1.

A. Elastic-perfectly-plastic

2.

B. Deviatoric stress

3.

D. 149.6 MPa

SOLUTION 3

Axial stress

$$\sigma_a = \frac{F}{A} = \frac{20000}{\pi \times (17.5^2 - 15^2)} = 78.4 \text{ MPa}$$

Torsional shear stress

$$\tau = \frac{Tr}{J} = \frac{32 \times 400 \times 1000 \times 17.5}{\pi(35^4 - 30^4)} = 103.2 \text{ MPa}$$

Centre of Mohr's circle given by:

$$C = \frac{\sigma_a}{2} = \frac{78.4}{2} = 39.2 \text{ MPa}$$

Radius of Mohr's circle, given by:

$$R = \tau_{max} = \sqrt{\left(\frac{\sigma_a}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{78.4}{2}\right)^2 + 103.2^2} = 110.4 \text{ MPa}$$

Max principal stress given by:

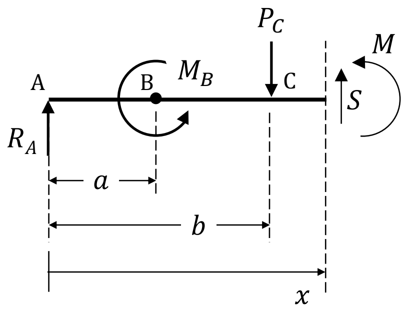
$$\sigma_1 = C + R = 39.2 + 110.4 = \mathbf{149.6\ MPa}$$

4.

B. $$EI\frac{d^2y}{dx^2} = R_A x - M_B\langle x-a\rangle^0 - P_C\langle x-b\rangle$$

SOLUTION 4

Sectioning the beam after the last discontinuity (taking origin at left hand side of the beam) and drawing a Free Body Diagram of left-hand side of section:



Taking moments about the section position (and applying Macauley's convention):

$$M + P_C\langle x-b\rangle + M_B\langle x-a\rangle^0 = R_A x$$

$$\therefore M = R_A x - P_C\langle x-b\rangle - M_B\langle x-a\rangle^0$$

Substituting this into the deflection of beams equation:

$$\boldsymbol{EI\frac{d^2y}{dx^2} = R_A x - M_B\langle x-a\rangle^0 - P_C\langle x-b\rangle}$$

5.

A. $y = \frac{1}{EI}\left(\frac{R_A x^3}{6} - \frac{M_B\langle x-a\rangle^2}{2} - \frac{P_C\langle x-b\rangle^3}{6} + Ax + B\right)$

SOLUTION 5

Integrating $EI\frac{d^2y}{dx^2} = R_A x - M_B\langle x-a\rangle^0 - P_C\langle x-b\rangle$ with respect to x gives:

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - M_B\langle x-a\rangle - \frac{P_C\langle x-b\rangle^2}{2} + A$$

Integrating again with respect to x:

$$EIy = \frac{R_A x^3}{6} - \frac{M_B\langle x-a\rangle^2}{2} - \frac{P_C\langle x-b\rangle^3}{6} + Ax + B$$

Rearranging this for y:

$$\boldsymbol{y = \frac{1}{EI}\left(\frac{R_A x^3}{6} - \frac{M_B\langle x-a\rangle^2}{2} - \frac{P_C\langle x-b\rangle^3}{6} + Ax + B\right)}$$

6.

A. OA

7.

C. 43.5 °C

SOLUTION 7

$$c_0 = \pi d_0 = \pi \times 15 = 47.12 \text{ mm}$$

$$c_1 = \pi d_1$$

$$c_1 - c_0 = \delta c = \pi \times 0.015 = 0.047 \text{ mm}$$

$$\delta c = c_0 \alpha \Delta T_{min}$$

$$\Delta T_{min} = \frac{\delta c}{c_0 \alpha} = \frac{0.047}{47.12 \times 23 \times 10^{-6}} = \mathbf{43.5\ °C}$$

8.

E. 25 MPa

SOLUTION 8

$$\sigma_{steel} = -\frac{A_{alu}}{A_{steel}} \frac{\Delta T(\alpha_{steel} - \alpha_{alu})}{\left[\frac{1}{E_{alu}} + \frac{A_{alu}}{A_{steel}E_{steel}}\right]} = -\frac{40 \times (11 \times 10^{-6} - 23 \times 10^{-6})}{\left[\frac{1}{70 \times 10^9} + \frac{100 \times 10^{-6}}{100 \times 10^{-6} \times 200 \times 10^9}\right]} = 2.4889 \times 10^7 \text{ Pa} = \mathbf{25\ MPa}$$

9.

A. Yes

SOLUTION 9

$$K_{max} = 1.33\sigma\sqrt{\pi a}$$

Therefore, if K_c and a_c, represent the critical values of stress intensity factor and crack length, respectively:

$$K_c = 1.33\sigma\sqrt{\pi a_c}$$

Rearranging,

$$a_c = \frac{\left(\frac{K_{I_c}}{1.33\sigma}\right)^2}{\pi} = \frac{\left(\frac{62}{1.33 \times 850}\right)^2}{\pi} = 0.000957 \text{ m} = 0.957 \text{ mm}$$

Since the crack exceeds the critical crack length, i.e. $1 \text{ mm} > 0.957 \text{ mm}$, **the component will fracture under this load**.

10.

E. Crack propagation

11.

B. 3.4 x 10^{-4} m

SOLUTION 11

$$\Delta L = L\alpha\Delta T$$

$$\therefore \Delta L = 1.25 \times 11 \times 10^{-6} \times 25 = \mathbf{3.4 \times 10^{-4}\ m}$$

12.

A. Yes

SOLUTION 12

$$\tau = \frac{Tr}{J} = \frac{32 \times 1600 \times 20 \times 10^{-3}}{\pi \times (40 \times 10^{-3})^4} = 127\ \mathrm{MPa}$$

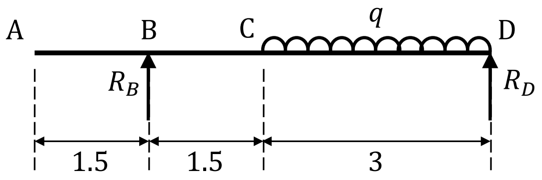
$$127 > \frac{\sigma_y}{2} = \frac{250}{2} = 125\ \mathrm{MPa}$$

13.

D. 4 kN

SOLUTION 13

Reaction forces will be present at positions B and E, namely R_B and R_E as shown below.



Taking moments about position B:

$$R_D \times 4.5 = q \times 3 \times \left(1.5 + \frac{3}{2}\right)$$

Substituting in $q = 2\text{ kN}$ and re-arranging:

$$\therefore \boldsymbol{R_D = 4\text{ kN}}$$

14.

C. 2 kN

SOLUTION 14

Vertical equilibrium:

$$R_B + R_D = q \times 3$$

Substituting in $q = 2\text{ kN}$ and $R_D = 4\text{ kN}$ and re-arranging:

$$\therefore \boldsymbol{R_B = 2\text{ kN}}$$

15.

E. E

SOLUTION 15

For cross-section A:

$$I = \frac{bd^3}{12} = \frac{200 \times 300^3}{12} = 450{,}000{,}000\text{ mm}^4$$

For cross-section B:

$$I = \frac{\pi\left({D_o}^4 - {D_i}^4\right)}{64} = \frac{\pi \times (300^4 - 200^4)}{64} = 319{,}068{,}003.9\text{ mm}^4$$

For cross-section C:

$$I = \frac{\pi D^4}{64} = \frac{\pi \times 250^4}{64} = 191{,}747{,}598.5\text{ mm}^4$$

For cross-section D:

$$I = \frac{bd^3}{12} = \frac{250^4}{12} = 325{,}520{,}833.3 \text{ mm}^4$$

For cross-section E:

$$I = \frac{b_o {d_o}^3 - b_i {d_i}^3}{12} = \frac{250 \times 350^3 - 150 \times 250^3}{12} = \mathbf{697,916,666.7\ mm^4}$$

Cross-section E has the largest 2nd moment of area.

16.

C. 345.3 mm

SOLUTION 16

2nd moment of area of a solid circular cross-section:

$$I = \frac{\pi D^4}{64}$$

$$\therefore D = \sqrt[4]{\frac{64I}{\pi}}$$

Substituting in the largest 2nd moment of area from Q7 ($697{,}916{,}666.7 \text{ mm}^4$):

$$D = \sqrt[4]{\frac{64 \times 697{,}916{,}666.7}{\pi}} = \mathbf{345.3\ mm}$$

17.

C. Increasing R-ratio

18.

D. 1.4 MPa

SOLUTION 18

von Mises yield criterion states yield occurs when:

$$\sigma_y^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$$

For an internally pressurised cylinder

$$\sigma_1 = \sigma_\theta = \frac{pr}{t}$$

$$\sigma_2 = \sigma_A = \frac{pr}{2t}$$

Substituting in:

$$\sigma_y^2 = \frac{p^2r^2}{t^2} - \frac{p^2r^2}{2t^2} + \frac{p^2r^2}{4t^2}$$

$$\sigma_y^2 = \frac{3}{4}\frac{p^2r^2}{t^2}$$

$$p = \sqrt{\frac{4}{3}}\frac{\sigma_y\, t}{r}$$

$$p = \sqrt{\frac{4}{3}}\frac{250 \times 10^{-6} \times 3 \times 10^{-3}}{0.625} = \mathbf{1.4\ MPa}$$

19.

A. 74.3 MPa

SOLUTION 19

Bending stress

$$\sigma_b = \frac{My}{I} = \frac{64 \times 20 \times 50 \times 10^{-3} \times 2.5 \times 10^{-3}}{\pi \times (5 \times 10^{-3})^4} = 81.5\ \text{MPa}$$

Torsional shear stress

$$\tau = \frac{Tr}{J} = \frac{32 \times 20 \times 75 \times 10^{-3} \times 2.5 \times 10^{-3}}{\pi \times (5 \times 10^{-3})^4} = 61.1 \text{ MPa}$$

Radius of Mohr's circle, given by:

$$R = \tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{81.5}{2}\right)^2 + 61.1^2} = \mathbf{73.4\ MPa}$$

20.

E. 126.6 MPa

SOLUTION 20

Given $\sigma_x = 50$ MPa, $\sigma_y = 100$ MPa and $\tau_{xy} = 45$ MPa:

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 100}{2} = 75.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{z\theta}^2} = \sqrt{\left(\frac{50 - 100}{2}\right)^2 + 45^2} = 51.5 \text{ MPa}$$

$$\sigma_1 = C + R = 75 + 51.5 = \mathbf{126.5\ MPa}$$